Section 4.5 (page 306)

$\int f(g(x))g'(x)\,dx$	$u = g(x)$	$du = g'(x)\,dx$
1. $\int (8x^2 + 1)^2(16x)\,dx$	$8x^2 + 1$	$16x\,dx$
3. $\int \frac{x}{\sqrt{x^2 + 1}}\,dx$	$x^2 + 1$	$2x\,dx$
5. $\int \tan^2 x \sec^2 x\,dx$	$\tan x$	$\sec^2 x\,dx$

7. No **9.** Yes **11.** $\frac{1}{5}(1 + 6x)^5 + C$
13. $\frac{2}{3}(25 - x^2)^{3/2} + C$ **15.** $\frac{1}{12}(x^4 + 3)^3 + C$
17. $\frac{1}{15}(x^3 - 1)^5 + C$ **19.** $\frac{1}{3}(t^2 + 2)^{3/2} + C$
21. $-\frac{15}{8}(1 - x^2)^{4/3} + C$ **23.** $1/[4(1 - x^2)^2] + C$
25. $-1/[3(1 + x^3)] + C$ **27.** $-\sqrt{1 - x^2} + C$
29. $-\frac{1}{4}(1 + 1/t)^4 + C$ **31.** $\sqrt{2x} + C$

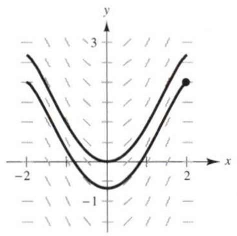
33. $\frac{2}{5}x^{5/2} + \frac{10}{3}x^{3/2} - 16x^{1/2} + C = \frac{1}{15}\sqrt{x}(6x^2 + 50x - 240) + C$

35. $\frac{1}{4}t^4 - 4t^2 + C$

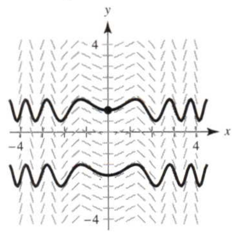
37. $6y^{3/2} - \frac{2}{5}y^{5/2} + C = \frac{2}{5}y^{3/2}(15 - y) + C$

39. $2x^2 - 4\sqrt{16 - x^2} + C$ **41.** $-1/[2(x^2 + 2x - 3)] + C$

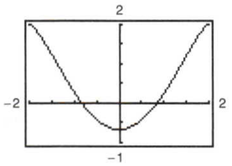
43. (a) Answers will vary. Example:



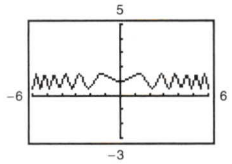
45. (a) Answers will vary. Example:



43. (b) $y = -\frac{1}{3}(4 - x^2)^{3/2} + 2$



45. (b) $y = \frac{1}{2}\sin x^2 + 1$



47. $-\cos(\pi x) + C$ **49.** $-\frac{1}{4}\cos 4x + C$ **51.** $-\sin(1/\theta) + C$

53. $\frac{1}{4}\sin^2 2x + C$ or $-\frac{1}{4}\cos^2 2x + C_1$ or $-\frac{1}{8}\cos 4x + C_2$

55. $\frac{1}{5}\tan^5 x + C$ **57.** $\frac{1}{2}\tan^2 x + C$ or $\frac{1}{2}\sec^2 x + C_1$

59. $-\cot x - x + C$ **61.** $f(x) = 2\cos(x/2) + 4$

63. $f(x) = -\frac{1}{2}\cos 4x - 1$ **65.** $f(x) = \frac{1}{12}(4x^2 - 10)^3 - 8$

67. $\frac{2}{5}(x + 6)^{5/2} - 4(x + 6)^{3/2} + C = \frac{2}{5}(x + 6)^{3/2}(x - 4) + C$

69. $-\left[\frac{2}{3}(1 - x)^{3/2} - \frac{4}{5}(1 - x)^{5/2} + \frac{2}{7}(1 - x)^{7/2}\right] + C = -\frac{2}{105}(1 - x)^{3/2}(15x^2 + 12x + 8) + C$

71. $\frac{1}{8}\left[\frac{2}{5}(2x - 1)^{5/2} + \frac{4}{3}(2x - 1)^{3/2} - 6(2x - 1)^{1/2}\right] + C = \left(\sqrt{2x - 1}/15\right)(3x^2 + 2x - 13) + C$

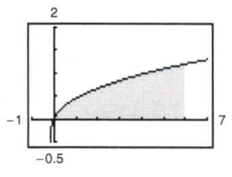
73. $-x - 1 - 2\sqrt{x + 1} + C$ or $-\left(x + 2\sqrt{x + 1}\right) + C_1$

75. 0 **77.** $12 - \frac{8}{9}\sqrt{2}$ **79.** 2 **81.** $\frac{1}{2}$ **83.** $\frac{4}{15}$ **85.** $3\sqrt{3}/4$

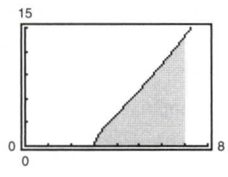
87. $f(x) = (2x^3 + 1)^3 + 3$ **89.** $f(x) = \sqrt{2x^2 - 1} - 3$

91. 1209/28 **93.** 4 **95.** $2\left(\sqrt{3} - 1\right)$

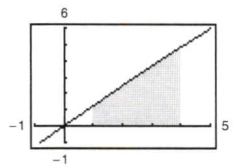
97. $\frac{14}{3}$



99. $\frac{144}{5}$



101. 9.21



103. $\frac{272}{15}$ **105.** $\frac{2}{3}$ **107.** (a) $\frac{64}{3}$ (b) $\frac{128}{3}$ (c) $-\frac{64}{3}$ (d) 64

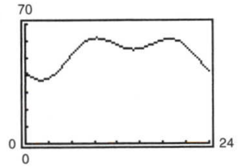
109. $2\int_0^3 (4x^2 - 6)\,dx = 36$

111. If $u = 5 - x^2$, then $du = -2x\,dx$ and $\int x(5 - x^2)^3\,dx = -\frac{1}{2}\int(5 - x^2)^3(-2x)\,dx = -\frac{1}{2}\int u^3\,du$.

113. 16 **115.** \$250,000

117. (a) Relative minimum: (6.7, 0.7) or July
Relative maximum: (1.3, 5.1) or February
(b) 36.68 in. (c) 3.99 in.

119. (a)



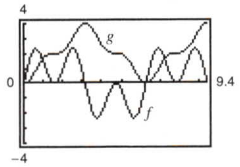
Maximum flow: $R \approx 61.713$ at $t = 9.36$.

(b) 1272 thousand gallons

121. (a) $P_{0.50,\,0.75} \approx 35.3\%$ (b) $b \approx 58.6\%$

123. (a) \$9.17 (b) \$3.14

125. (a)

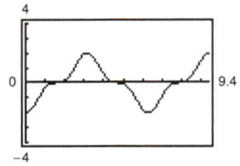


(b) g is nonnegative because the graph of f is positive at the beginning, and generally has more positive sections than negative ones.

(c) The points on g that correspond to the extrema of f are points of inflection of g.

(d) No, some zeros of f, such as $x = \pi/2$, do not correspond to extrema of g. The graph of g continues to increase after $x = \pi/2$ because f remains above the x-axis.

(e)



The graph of h is that of g shifted 2 units downward.

127. (a) Proof (b) Proof

129. False. $\int (2x + 1)^2\,dx = \frac{1}{6}(2x + 1)^3 + C$

131. True **133.** True **135–137.** Proofs

139. Putnam Problem A1, 1958